PHYSICS

DPP

DAILY PRACTICE PROBLEMS

DPP No. 63

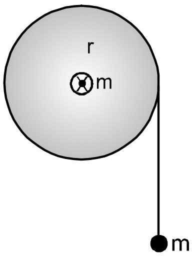
Total Marks : 27

Max. Time : 29 min.

Topics : Rigid Body Dynamics, Work, Power and Energy, Circular Motion, Center of Mass

Type of Questions		M.M., Min.
Single choice Objective ('–1' negative marking) Q.1 Q.2	(3 marks, 3 min.)	[6, 6]
Multiple choice objective ('–1' negative marking) Q.3	(4 marks, 4 min.)	[4, 4]
Subjective Questions ('–1' negative marking) Q.4 to Q. 5	(4 marks, 5 min.)	[8, 10]
Comprehension ('–1' negative marking) Q.6 to Q.8	(3 marks, 3 min.)	[9, 9]

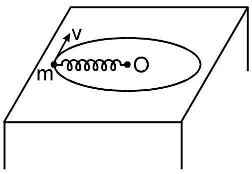
1. A uniform disc of mass m and radius r and a point mass m are arranged as shown in the figure. The acceleration of point mass is: (Assume there is no slipping between pulley and thread and the disc can rotate smoothly about a fixed horizontal axis passing through its centre and perpendicular to its plane)



(A) $\frac{g}{2}$ (B) $\frac{g}{3}$

(C) $\frac{2g}{3}$ (D) none of these

2. Mass m is connected with an ideal spring of natural length ℓ whose other end is fixed on a smooth horizontal table. Initially spring is in its natural length ℓ. Mass m is given a velocity 'v' perpendicular to the spring and released. The velocity perpendicular to the spring when its length is ℓ + x, will be



(A) $\frac{2v\ell}{\ell + x}$ (B) $\frac{2v^2\ell}{\ell + x}$

(C) $\frac{v\ell}{\ell + x}$ (D) zero

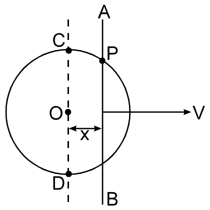
3. A ball of mass m is attached to the lower end of a light vertical spring of force constant k. The upper end of the spring is fixed. The ball is released from rest with the spring at its normal (unstretched) length, and comes to rest again after descending through a distance x.

(A) $x = \frac{mg}{k}$ (B) $x = \frac{2mg}{k}$

(C) the ball will have no acceleration at the position where it has descended through $\frac{x}{2}$

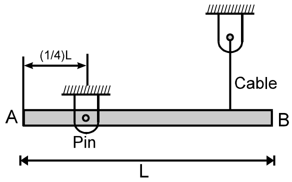
(D) the ball will have an upward acceleration equal to g at its lowermost position.

4. A rod AB is moving on a fixed circle of radius R with constant velocity 'v' as shown in figure. P is the point of intersection of the rod and the circle. At an instant the rod is at a distance $x = \frac{3R}{5}$ from centre of the circle. The velocity of the rod is perpendicular to the rod and the rod is always parallel to the diameter CD.



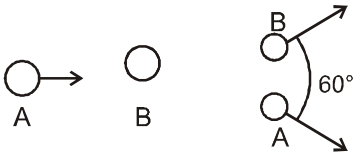
(a) Find the speed of point of intersection P.
(b) Find the angular speed of point of intersection P with respect to centre of the circle.

5. A uniform beam of length L and mass 'm' is supported as shown. If the cable suddenly breaks, determine; immediately after the release.
(a) the acceleration of end B.
(b) the reaction at the pin support.



COMPREHENSION

A smooth ball 'A' moving with velocity 'V' collides with another smooth identical ball at rest. After collision both the balls move with same speed with angle between their velocities 60°. No external force acts on the system of balls.



6. The speed of each ball after the collision is

(A) $\frac{V}{2}$ (B) $\frac{V}{3}$ (C) $\frac{V}{\sqrt{3}}$ (D) $\frac{2V}{\sqrt{3}}$

7. If the kinetic energy lost is fully converted to heat then heat produced is

(A) $\frac{1}{3}mV^2$ (B) $\frac{2}{3}mV^2$ (C) 0 (D) $\frac{1}{6}mV^2$

8. The value of coefficient of restitution is

(A) 1 (B) $\frac{1}{3}$ (C) $\frac{1}{\sqrt{3}}$ (D) 0

Answers Key

1. (C) **2.** (C) **3.** (B)(C)(D)

4. (a) $V_P = \frac{5}{4} V$ (b) V cosec θ

5. (a) $\frac{9g}{7} \downarrow$ (b) $\frac{4w}{7} \uparrow$ **6.** (C) **7.** (D)

8. (B)

Hint & Solutions

2. since torque about O is zero,

angular momentum of mass m is conserved

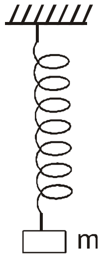
$$\therefore \quad m v\ell = m v_\perp (\ell + x) \; ; \; v_\perp = \frac{v\ell}{\ell + x}$$

3. initial velocity = final velocity = 0

from energy conservation

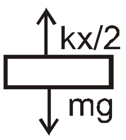
$$mgx - \frac{1}{2} kx^2 = 0$$

$$x = \frac{2mg}{k}$$



(at Natural length)

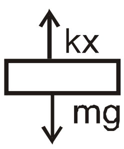
at deseended length = $\frac{x}{2}$



$$\frac{kx}{2} = k \,.\, \frac{2mg}{2k} = mg$$

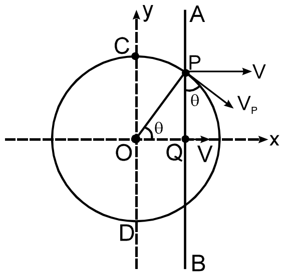
Net force = 0

$\Rightarrow$ a = 0 at lower most position



$$\text{force} = Kx - mg = K\frac{2mg}{K} - mg = mg$$

4.



As a rod AB moves, the point 'P' will always lie on the circle.

∴ its velocity will be along the circle as shown by 'V_P' in the figure. If the point P has to lie on the rod 'AB' also then it should have component in 'x' direction as 'V'.

∴ $V_P \sin\theta = V$

⇒ $V_P = V \operatorname{cosec}\theta$

here $\cos\theta = \frac{x}{R} = \frac{1}{R} \cdot \frac{3R}{5} = \frac{3}{5}$

∴ $\sin\theta = \frac{4}{5}$ ∴ $\operatorname{cosec}\theta = \frac{5}{4}$ ∴ $V_P = \frac{5}{4} V$

...Ans.

$\omega = \frac{V_P}{R} = \frac{5V}{4R}$

ALTERNATIVE SOLUTION :

Sol. (a) Let 'P' have coordinate (x, y)

$x = R\cos\theta,\ y = R\sin\theta.$

$V_x = \frac{dx}{dt} = -R\sin\theta \frac{d\theta}{dt} = V \Rightarrow \frac{d\theta}{dt} = \frac{-V}{R\sin\theta}$

$V_Y = R\cos\theta \frac{d\theta}{dt} = R\cos\theta\left(-\frac{V}{R\sin\theta}\right)$

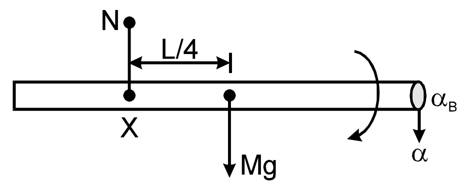
$= -V\cot\theta$

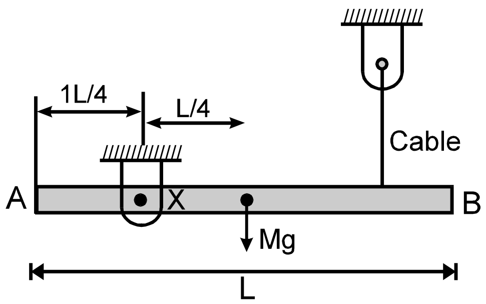
∴ $V_P = \sqrt{V_x^2 + V_y^2} = \sqrt{V^2 + V^2\cot^2\theta}$

$= V\operatorname{cosec}\theta$ **...Ans.**

Sol. (b) $\omega = \frac{V_P}{R} = \frac{5V}{4R}$

5. Taking torques w.r.t 'x'





M.I. of the rod w.r.t axis of rotation

$$I_x = I_{cm} + \frac{ML^2}{16}$$

$$= \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$$

(i) $Mg.\frac{L}{4} = I.\alpha$

$$Mg.\frac{L}{4} = \frac{7ML^2}{48}.\alpha$$

$$\Rightarrow \alpha = \frac{12g}{7L} \qquad a_B = Ra$$

$$= \frac{3L}{4}.\frac{12g}{7L} = \frac{9g}{7}\downarrow$$

(ii) $a_{cm} = \frac{L}{4}.\alpha = \frac{3g}{7}$

also apply equation of motion on COM

$$Mg - N = M.\frac{3g}{7}$$

3Mg 4Mg

[Ans.: (a) $\frac{9g}{7}\downarrow$ (b) $\frac{4w}{7}\uparrow$]

6.to 8 From conservation of momentum

$$mv = mv'\cos 30° + mv'\cos 30°$$

$$\therefore v' = \frac{v}{2\cos 30°} = \frac{v}{\sqrt{3}}$$

7. Loss in kinetic energy

$$= \frac{1}{2}mv^2 - 2\times\frac{1}{2}m\left(\frac{v}{\sqrt{3}}\right)^2 = \frac{1}{6}mv^2$$

8. Initially B was at rest, therefore line of impact is along final velocity of B.

$$\therefore e = \frac{v' - v'\cos 60°}{v\cos 30} = \frac{\frac{1}{2}\frac{v}{\sqrt{3}}}{v\times\frac{\sqrt{3}}{2}} = \frac{1}{3}$$